

Application of A New Cohesive Zone Model in Low Cycle Fatigue

Xuan Cao^{*1}; Michael Vormwald²

^{1,2} Material Mechanics Group, Technische Universität Darmstadt Petersenstraße 12, 64287 Darmstadt, Germany

^{*1}cao@wm.tu-darmstadt.de; ²vormwald@wm.tu-darmstadt.de

Abstract

A cohesive zone model (CZM) for material fatigue analysis is presented. This cyclic CZM contains a damage variable and a corresponding damage evolution law. Considering the influence of the constraint condition for CZM, the triaxiality factor is also involved in this model. The new cyclic CZM can describe the different damage evolution mechanisms in the fatigue crack initiation and fatigue crack growth process. A unique set of model parameters is used. The material damage parameters included in the damage evolution law can be obtained from the material strain-life curve. Then they are used for fatigue crack growth calculation. The simulation results are compared with the experimental data in the very low cycle fatigue regime.

Keywords

Cohesive Zone Model; Damage Evolution; Triaxiality Dependence; Low Cycle Fatigue

Introduction

In reality, numerous engineering structures and components are operated under cyclic loading. Fatigue fracture of materials and structures is a failure phenomenon which can cause catastrophes. Therefore, the fatigue life prediction and fatigue crack growth analysis are very important issues for material science and engineering. Fracture mechanics concepts can be used to solve such problems but modeling expenses might become high. Alternative approaches need to be introduced in this field, and the selected method should be simple enough in theoretical understanding and practical application. Synthesizing all the consideration, the application of the cohesive zone models in fatigue calculation is a promising development.

Cohesive zone models (CZM) describe the material damage in a narrow strip zone. Originally, the idea of the CZM was proposed by Dugdale (1960) and Barenblatt (1962). In 1976, the CZM was firstly applied by Hillerborg (1976) in finite element calculations for concrete material. From then on, the CZM was used

widely in the finite element simulation for material fracture. It was applied by Needleman (1987, 1990) for modeling inclusion debonding and interface decohesion in microscopic and macroscopic scales, respectively. Subsequently, it was employed by Tvergaard and Hutchinson (1992) for crack growth calculation in elastic-plastic solids. In the last two decades, the CZM was improved by many researchers and scientific groups (Chaboche et al. 1997, Yuan et al. 1996, Cornec et al. 2003, Scheider et al. 2003, Scheider et al. 2006, Mohammed et al. 2000, Chandra et al. 2002, Chen et al. 2003, Song et al. 2006). Now it is a robust tool for material fracture analysis.

However, a remarkable fact is that all these CZMs were just used for material fracture under monotonic loading. In the late 1990's, the CZM was firstly extended by De-Andrés et al. (1999) for fatigue crack growth simulation. A damage variable was introduced in this CZM to describe the cohesive element degradation. But the formulation did not contain a description about the damage evolution. Soon afterwards, Yang et al. (2001) developed a CZM based on a line spring model to simulate fatigue crack growth. The damage variable and the corresponding damage evolution were intrinsically involved in the fatigue process. However, in this model the unloading and reloading stiffness had different definitions. These models were not convenient and accurate for application. Then the cyclic CZM proposed by Roe and Siegmund (2003) explicitly introduced a damage variable and a damage evolution equation. The stiffness and maximum cohesive traction degraded with the damage variable. The unloading and reloading path used the same definition. The numerical calculation results of this model have reproduced many basic characteristics which are similar to the experimental phenomena in fatigue fracture. Nevertheless, there was no experimental validation for the damage evolution process and various other important factors for CZM were not considered, such as the influences of surrounding

continuum element, element friction and constraint condition. Based on Roe and Siegmund's (2003) method, some authors published further improvements and extensions (Abdul-Baqi et al. 2005, Bouvard et al. 2009, Xu et al. 2009). In addition, others also proposed various damage evolution hypotheses of CZMs for fatigue simulation (Ural et al. 2009, Pironi et al. 2011, Turon et al. 2007, Munoz et al. 2006, Beaurepaire et al. 2011). Recently, Jha and Banerjee (2012) presented a CZM for fatigue failure in complex stress states. The description of the damage and damage evolution were still from Roe and Siegmund's method (2003). This new CZM can consider the influence of various triaxiality conditions. But in this model, triaxiality was not involved in the damage evolution law. The work just focused on fatigue life analysis. Fatigue crack growth has not been calculated. Furthermore, the simulation results just gave some fatigue fracture trends, but no comparison with experimental results was included.

Cohesive Zone Model Approach

Fundamentals

The CZM intends to describe the real physical fracture process by phenomenological equations. The CZM can be adopted for a material without or with a crack. The total material damage is supposed to occur in the cohesive zone, the continuum elements in the vicinity are damage free. In practice, the CZM can be applied for any material, any fracture mode or any model dimension. In the CZM, there is a constitutive relationship between cohesive traction T and material separation δ , called traction-separation law (TSL). The TSL represents the material deterioration occurring in the interface under the monotonic loading process. In practical application, several TSLs can be chosen, for example, the cubic shape (Needleman 1987), the exponential shape (Needleman 1990), the trilinear shape (Tvergaard and Hutchinson 1992) and the partly constant shape (Scheider et al. 2003). All the TSLs contain two parameters, the maximum traction T_0 and the critical separation δ_0 . The area under the TSL represents the cohesive energy Γ_0 . It can be derived as $\Gamma_0 = \int_0^{\delta_0} T(\delta) d\delta$. If the shape of the TSL is selected, the cohesive parameters should be determined from experiments. Hence the cohesive parameters are both model and material parameters. In this paper, the TSL from Scheider et al. (2003) is taken as a basis (Fig. 1). More details about applying this CZM can be found in references (Scheider et al. 2003, Scheider et al. 2006).

$$T(\delta) = T_0 f(\delta) = T_0 * \begin{cases} 2\left(\frac{\delta}{\delta_1}\right) - \left(\frac{\delta}{\delta_1}\right)^2 & \delta < \delta_1 \\ 1 & \delta_1 \leq \delta \leq \delta_2 \\ 2\left(\frac{\delta - \delta_2}{\delta_0 - \delta_2}\right)^3 - 3\left(\frac{\delta - \delta_2}{\delta_0 - \delta_2}\right)^2 + 1 & \delta_2 < \delta \leq \delta_0 \end{cases} \quad (1)$$

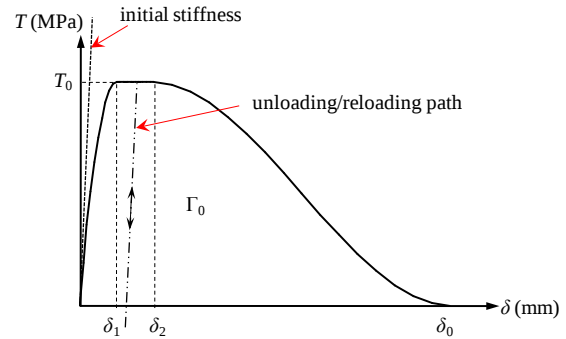


FIG. 1 TRACTION SEPARATION LAW ACCORDING TO SCHEIDER ET AL. (2003), GIVEN BY EQUATION (1)

Advanced Property - Triaxiality Dependent Behaviour

Besides using the CZM in a phenomenological way, it is possible to find a relationship between the micromechanics of a fracture process and the TSL. In micromechanics, the material ductile fracture is represented by void nucleation, growth and final coalescence. Based on this process, a micromechanical model for material ductile fracture was proposed by Gurson (1977), later extended by Tvergaard and Needleman (1984), which is now called GTN model. Using the GTN model to simulate a tension test for one cell element, a stress displacement curve can be derived. Such a curve can be directly used as a TSL, or can be fitted to another common shape, which might be implemented in a finite element code. For this micromechanical traction separation curve, some investigators found that it was strongly influenced by the triaxiality conditions. Here, the triaxiality value H means the ratio between the hydrostatic stress σ_h and the von Mises equivalent stress σ_{Mises} .

$$H = \frac{\sigma_h}{\sigma_{Mises}} \quad (2)$$

Siegmund and Brocks were the first researchers to propose such a triaxiality dependent CZM (1999, 2000). More investigations on triaxiality dependent behaviour were published by Anvari et al. (2006), Scheider (2009) and Banerjee et al. (2009). The research conclusions of all authors can be summarized: The cohesive parameters T_0 , δ_0 and Γ_0 are not pure material constants but dependent on the triaxiality conditions. With increasing triaxiality, the maximum traction T_0

increases, but the cohesive energy Γ_0 (or critical separation δ_0) decreases.

When the triaxiality dependent CZM is applied in finite element simulations, another important issue is how to get the triaxiality value for a cohesive element. In cohesive elements, there are no definitions about hydrostatic stress and von Mises equivalent stress. So the triaxiality can only be obtained from the adjacent continuum element and has to be transferred to the cohesive element.

For describing the triaxiality dependent behavior, equation (3) is used. This equation is taken from the literature (Scheider 2009) and a small modification is introduced.

$$\begin{aligned} T_0 / \sigma_Y &= 3.3 - 4.8 \exp\left(\frac{0.1 - H}{0.45}\right) \\ \delta_0 / D_{cell} &= \frac{1}{5} \left[0.138 + 0.635 \exp\left(\frac{0.858 - H}{0.788}\right) \right] + 0.5346 \\ \Gamma_0 &= \frac{23}{40} \delta_c T_0 \end{aligned} \quad (3)$$

T_0 , δ_0 and Γ_0 are the cohesive parameters. They are all triaxiality dependent. σ_Y is the material yielding stress. D_{cell} is an important parameter in a GTN simulation. It means the element height. The modification concerns the second part of equation (3), i.e. the calculation of δ_0 . A factor 1/5 is introduced because of convergence problems. In reference (Scheider 2009) the examined material is S460N, the yield stress is $\sigma_Y = 470\text{MPa}$ and the element height is $D_{cell} = 0.1\text{mm}$. Also H has a range boundary ($1 \leq H \leq 4$). If the triaxiality value jumps out of the range, the cohesive parameters are to be calculated under conditions of $H = 1$ or $H = 4$, respectively.

Numerical Model

Overview

The conventional TSLs cannot describe the cyclic degradation for the cohesive element. In a trial to overcome this defect, in this paper, a tentative idea from Roe and Siegmund (2003) is taken as a starting point. Consulting the damage theory of the continuum element, a damage variable and a corresponding damage evolution law are introduced in the CZM following ideas of Scheider et al. (2003). This cyclic CZM is applied for fatigue analysis.

There are two main fatigue phenomena – fatigue crack initiation and fatigue crack growth. In engineering

practice, fatigue crack initiation is usually investigated using smooth unnotched specimens and fatigue crack growth is measured using pre-cracked specimens. The triaxiality conditions for these two standard specimens are totally different. Realistic modelling of the fatigue phenomena therefore requires taking the triaxiality into account in the damage evolution process.

A triaxiality dependent cyclic CZM is proposed in this paper. The model should be used to reproduce the experimental fatigue phenomena which contain fatigue crack initiation and fatigue crack growth, just using a unique set of parameters. For simplicity, in the proposed CZM only mode-I fracture is considered. This means in CZM, just normal separation occurs.

Construction of the Cyclic CZM

With the damage variable D , the maximum cohesive traction and normal stiffness of cohesive elements will degrade.

$$\tilde{T}_0^N = \left[T_{0(H)}^N \right] (1 - D) \quad (4)$$

$$\tilde{k}_N = k_N (1 - D) = \frac{2T_{0(H)}^N}{\delta_1} (1 - D) = \frac{2\tilde{T}_0^N}{\delta_1} \quad (5)$$

T_0^N and $\tilde{T}_0^N$ are the initial and current maximum cohesive traction. The subscript (H) represents that the maximum cohesive traction is dependent on the triaxiality conditions. k_N and $\tilde{k}_N$ are the initial and current normal stiffness of the cohesive element. δ_1 is a shape parameter for the TSL and relates to the critical separation δ_0 .

For the triaxiality dependent cyclic CZM, it is difficult to judge the loading and unloading conditions because the separation δ and the critical separation δ_0 alter simultaneously. So, a relative normal separation δ_{re}^N and its increment $\Delta\delta_{re}^N$ are defined as

$$\delta_{re}^N = \frac{\delta^N}{\delta_{0(H)}^N} \quad (6)$$

$$\Delta\delta_{re}^N = \frac{\delta_t^N}{\delta_{0(H_t)}^N} - \frac{\delta_{t-\Delta t}^N}{\delta_{0(H_{t-\Delta t})}^N} = \delta_{re}^N(t) - \delta_{re}^N(t-\Delta t) \quad (7)$$

δ^N is the absolute normal separation. δ_0^N is the critical normal separation. The subscript (H) represents that the critical separation is triaxiality dependent. The subscripts $t-\Delta t$ and t represent the previous and

current time increment. $\delta_{t-\Delta t}^N$ and δ_t^N are the absolute normal separation in the time increment $t-\Delta t$ and t . $\delta_{0(H_{t-\Delta t})}^N$ and $\delta_{0(H_t)}^N$ are the critical normal separations in the time increment $t-\Delta t$ and t . In the different time increments if the triaxiality value does not change, these two critical separations should be the same. $\delta_{re(t-\Delta t)}^N$ and $\delta_{re(t)}^N$ are the relative normal separation in the time increment $t-\Delta t$ and t .

Based on the definition of $\Delta\delta_{re}^N$, the loading and unloading conditions are summarized as follow. A loading step is indicated by $\Delta\delta_{re}^N > 0$, and an unloading step is indicated by $\Delta\delta_{re}^N < 0$.

The response of the cohesive element in the first loading step should comply with the monotonic TSL, which is defined in equation (1). The following unloading and reloading process use the same equation, a linear function

$$T_t^N = T_{t-\Delta t}^N + \tilde{k}_N [\Delta\delta_{re}^N * \delta_{0(H_t)}^N] \quad (8)$$

$T_{t-\Delta t}^N$ and T_t^N are the normal traction in the different time increment.

In the previous loading process a maximum relative normal separation $\delta_{re\max}^N$ may be reached. During the reloading period, if the current relative normal separation exceeds this maximum value, the reloading equation uses a new definition

$$T^N = \begin{cases} \tilde{T}_0^N \left[2 \left(\frac{\delta^N}{\delta_1} \right) - \left(\frac{\delta^N}{\delta_1} \right)^2 \right] & \delta^N \leq \delta_1 \\ \tilde{T}_0^N & \delta^N > \delta_1 \end{cases} \quad (9)$$

T^N and δ^N are the normal traction and separation in the current calculation increment.

During the unloading/compression process, the normal separation may become negative value. At this moment a penalized stiffness should be adopted. But in the current cohesive model, the initial stiffness in normal direction is very high. So, application of the penalized stiffness is not necessary.

In order to determine the current damage state, a damage evolution law should be included in the cyclic CZM. Here, a new damage evolution equation is proposed as

$$\dot{D} = \left(\frac{A^{\frac{1}{H}} e^{(\lambda-H)}}{H} \right) \Delta\delta_{re}^N \left[\frac{T^N}{T_{0(H)}^N (1-D)} - \frac{\sigma_e}{T_{0(H)}^N} \right]^m \quad \dot{D} \geq 0 \quad (10)$$

$\dot{D}$ is the damage evolution. It is always a positive value. σ_e is the material endurance limit. H is the triaxiality value. A and m are the material dependent damage controlling parameters. λ is the material dependent triaxiality influence coefficient.

After the cohesive element is broken, in the unloading or compression period in order to avoid the adjacent continuum element penetrating each other, a contact condition should be considered. Here, when $\delta^N < 0$, the contact is defined as

$$T^N = k_N \delta^N \quad (11)$$

The application of the triaxiality dependent behaviour in cyclic CZM is a crucial aspect. The triaxiality dependent behaviour obtained from GTN simulation is just proper for monotonic loading condition. If this behaviour is applied for cyclic loading, the first problem is how to consider the triaxiality influence in unloading and reloading process. Actually, in the unloading/reloading process, it is still an open topic how the triaxiality influences the cohesive parameters. So, in this paper, a relative simple method is adopted. The triaxiality value keeps constant in the unloading process and a part of the reloading process. This part of the reloading process is also defined by equation (9), when the current relative normal separation does not exceed the maximum value $\delta_{re\max}^N$, which is obtained from the previous loading step. In this case the triaxiality does not change. Once $\delta_{re\max}^N$ is exceeded, the triaxiality updates again.

A schematic illustration for the cyclic process is shown in Fig. 2.

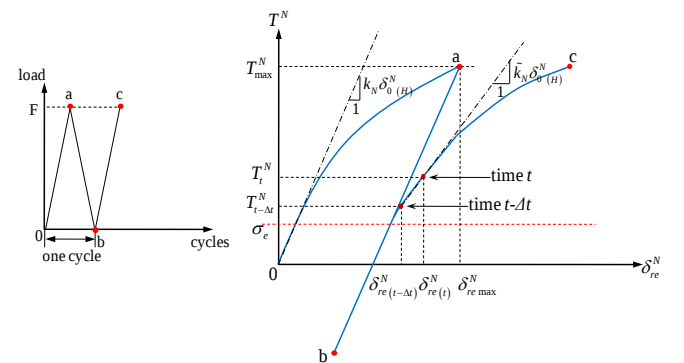


FIG. 2 SCHEMATIC ILLUSTRATION FOR CYCLIC PROCESS OF CZM

The new cyclic CZM is contained in a user defined subroutine, and it is connected with the commercial finite element program ABAQUS. The fatigue damage can be calculated for every time increment.

Model Verification

Material Behaviour and Damage Parameters Analysis

The triaxiality dependent cyclic CZM has been proposed and this new model should be verified by numerical simulation. The simulations contain two parts – fatigue crack initiation and fatigue crack growth. The primary idea is to reproduce the experimental results in fatigue test. The material S460N is chosen for the study. The triaxiality dependent behaviour for the simulated material is taken from reference (Scheider 2009). Some basic mechanical properties of this material are shown in Table 1.

TABLE 1 MECHANICAL PROPERTIES FOR MATERIAL S460N (BOLLER AND SEEGER 1987)

Material	$E(\text{MPa})$	$\sigma_Y(\text{MPa})$	$R_m(\text{MPa})$	$K'(\text{MPa})$	n'	$\sigma_e(\text{MPa})$
S460N	208000	470	682	1181	0.161	312

E is the material's Young's modulus. σ_Y is the material yield stress. R_m is the material ultimate tensile strength. K' is the material cyclic hardening coefficient. n' is the material cyclic hardening exponent. σ_e is the material endurance limit.

Before applying the cyclic CZM, it is necessary to discuss the material behaviour of the continuum element. The finite element model consists of two plane strain elements connected by one cohesive element, called homogeneous model. The model is loaded by cyclic displacement, the loading ratio is $R=1$.

In order to reflect the real material response under cyclic loading, the material behavior for the continuum elements uses a nonlinear kinematic hardening model in ABAQUS. Three different ways are offered by ABAQUS to define the nonlinear kinematic hardening component, specifying the material parameters directly, specifying half-cycle test data or specifying test data from a stabilized cycle. In this paper, nonlinear kinematic hardening model is identified from the stabilized cyclic test.

The stabilized hysteresis loops for the response of the continuum element under three loading levels are depicted in Fig.3.

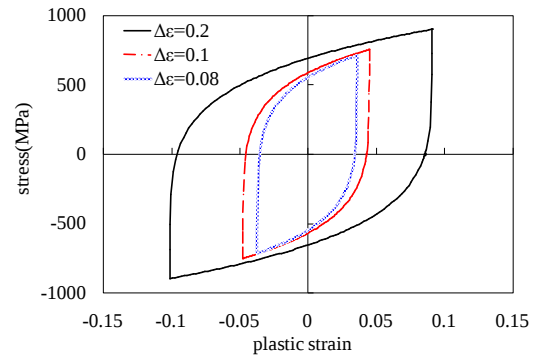


FIG. 3 STABILIZED HYSTERESIS LOOPS OF ONE CONTINUUM ELEMENT

For the chosen material, the unknown damage parameters in the damage evolution law are just A , m and λ . These parameters should be determined from the corresponding experiment test. However, it is impossible to discuss the influence of three damage parameters together. Hence, the material dependent triaxiality influence coefficient λ is fixed first to $\lambda=1.2$. The determination of this parameter is based on many trial calculations. The suggested λ value for material S460N ranges between 0.7 and 1.2, a change of the parameter λ will influence the other two parameters A and m . After λ is chosen, a simulation is applied to analyze the material dependent damage controlling parameters A and m . The calculation model is still the homogeneous model.

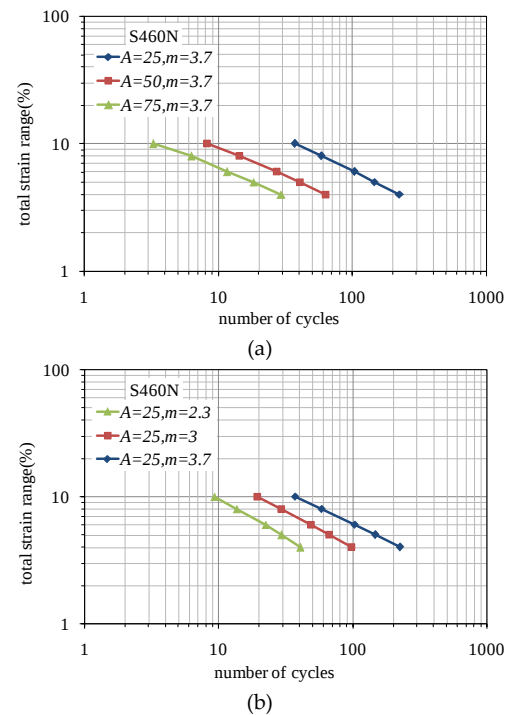


FIG. 4 EFFECT OF DAMAGE PARAMETERS A AND m IN DAMAGE EVOLUTION LAW

Series of the parameters A and m are investigated and

the calculation results are plotted in the material strain-life curves, shown in Fig. 4.

It is obvious that the parameter m influences the slope of the strain-life curve. With increasing the value of m , the slope of the strain-life curve becomes flatter. The parameter A does not influence the slope of the strain-life curve. However, it makes the curve moving parallelly. According to these rules, the parameters A and m can be identified by fitting the numerical results to the experimental strain-life curve.

Fatigue Crack Initiation Simulation

In practice, for fatigue crack initiation investigation usually an uncracked smooth specimen is used in the experiment. The detailed dimension and geometry of the specimen is depicted in Fig. 5. The surface of the specimen is mechanically polished. The loading condition in experiment employs total strain control and the strain ratio is -1. The failure criterion in experiment is crack initiation. When a 0.5mm length crack is detected in the surface of the middle section, the experiment is stopped.

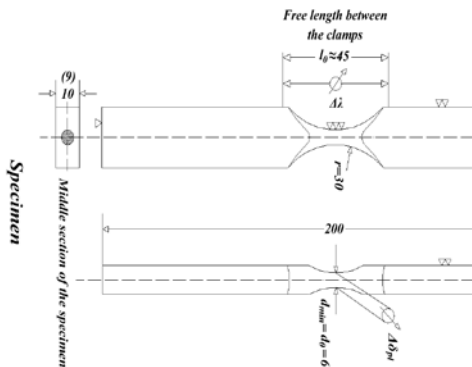


FIG. 5 SPECIMEN IN FATIGUE EXPERIMENT

TABLE 2 EXPERIMENTAL STRAIN-LIFE CURVE PARAMETERS (BOLLER AND SEEGER 1987)

Material	$E(\text{MPa})$	$\sigma'_f(\text{MPa})$	ϵ'_f	b	c
S460N	208000	1218	0.452	-0.104	-0.536

All the test results are plotted in Fig. 6 together with the regression curve

$$\epsilon_a = \epsilon_{a,el} + \epsilon_{a,pl} = \frac{\sigma'_f}{E} (2N)^b + \epsilon'_f (2N)^c \quad (12)$$

ϵ_a is the strain amplitude. $\epsilon_{a,el}$ and $\epsilon_{a,pl}$ are the elastic and plastic part for the strain amplitude. N is the fatigue crack initiation life. σ'_f is the fatigue strength coefficient, b is the fatigue strength exponent, ϵ'_f is the

fatigue ductility coefficient, c is the fatigue ductility exponent. Values are taken from reference (Boller and Seeger 1987) and shown in Table 2.

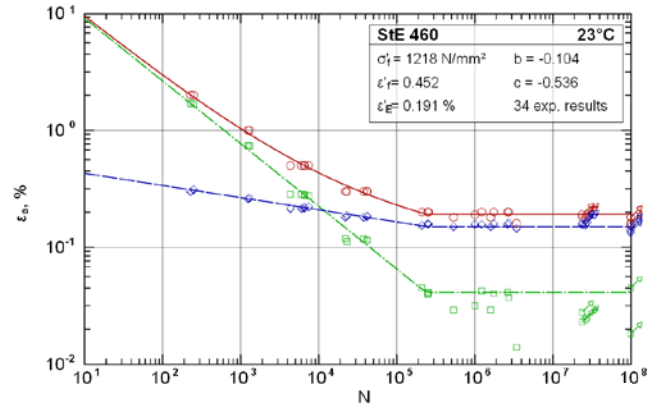


FIG. 6 EXPERIMENTAL STRAIN-LIFE CURVE FOR S460N (Vormwald 2011)

The simulation model is used here as described in the previous section. Two plane strain elements are connected by one cohesive element. When the cohesive element is broken, it means the crack initiation. Now the investigations just focus on the very low cycle fatigue regime, so the calculations are applied within 600 cycles. In the previous section, the analysis of the damage parameters has been explained. First, λ is fixed to $\lambda=1.2$. By fitting the simulation results to the experimental strain-life curve, the damage controlling parameters A and m can be determined. For material S460N, $A=25$ and $m=3.7$. The experiment and simulation results are plotted in Fig. 7. These parameters need further validation.

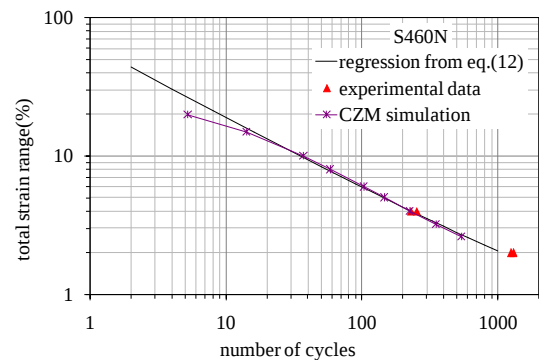


FIG. 7 COMPARISON OF STRAIN-LIFE CURVE FOR S460N

Fatigue Crack Propagation Simulation

As an important part of fatigue fracture, the fatigue crack growth simulation is applied to validate the damage parameters.

Fatigue crack growth is usually measured by pre-cracked specimens, such as CT specimen, MT specimen or DCB specimen. The triaxiality condition is

totally different from the fatigue crack initiation simulation. The triaxiality dependent cyclic CZM should be able to reflect this different mechanism successfully.

The experimental data of fatigue crack propagation on material S460N are shown in Fig. 8. It is clear that the experimental data can be expressed by Paris law. Now all the investigations focus on very low cycle fatigue regime, so just the experimental data at fast crack growth rates are used for comparison.

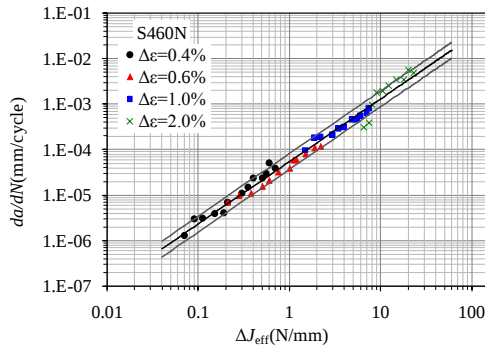


FIG. 8 EXPERIMENTAL FATIGUE CRACK GROWTH RATES CURVE FOR S460N (VORMWALD 2011)

The simulation model is a two dimensional compact tension specimen. The width of the model is 50mm and the height is 60mm. The initial crack length a_0 is 25mm and the ligament length is 25mm, too. The type of the continuum element is plane strain element. The cohesive elements locate along the ligament and the size of the single element is 0.125mm. The details for the finite element model are depicted in Fig. 9.

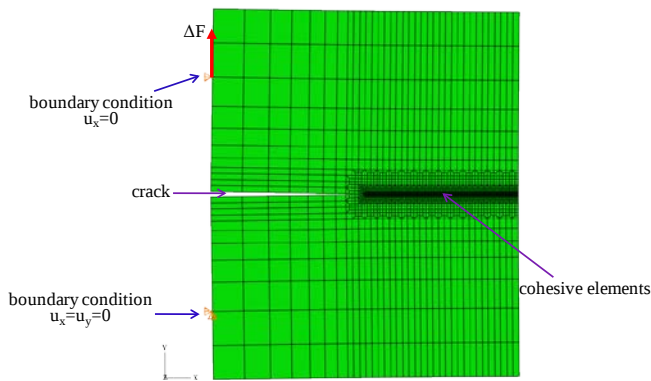


FIG. 9 FINITE ELEMENT MODEL FOR SIMULATED COMPACT TENSION SPECIMEN

In the simulation, three different cyclic loading ranges are used: $\Delta F=1200\text{N}$, $\Delta F=950\text{N}$ and $\Delta F=800\text{N}$. The loading ratio is $R=0$.

The response of the cohesive element needs an explanation. The CT model (in Fig.9) under loading range $\Delta F=1200\text{N}$ is discussed. The first cohesive element at the crack tip is selected. The traction

separation responses in two integration points of this cohesive element are plotted in Fig 10 and Fig11. In the following figures, the two integration points of the cohesive element are called IP1 and IP2. The point IP2 is closer to the crack tip.

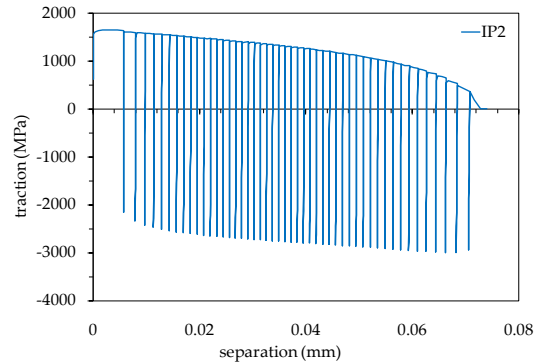


FIG. 10 TRACTION SEPARATION RESPONSE IN IP2

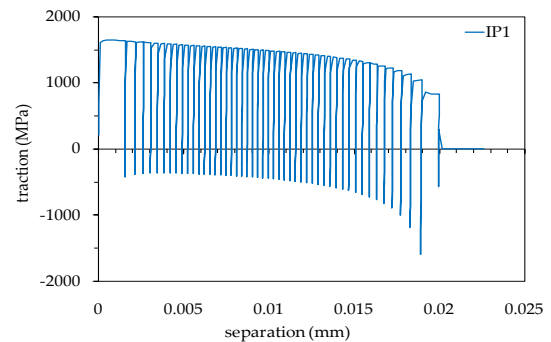


FIG. 11 TRACTION SEPARATION RESPONSE IN IP1

The traction separation responses in two integration points are very similar. After maximum traction T_0 in the cohesive element is reached, the cohesive traction in the later loading step always degrades. Until the final failure, the traction becomes zero. The stiffness degradation with the damage variable is not obvious because the initial stiffness of the cohesive element is very high. Until the final failure, the separation in IP2 is much bigger than the separation in IP1.

The damage evolution in the two integration points are showed in Fig.12.

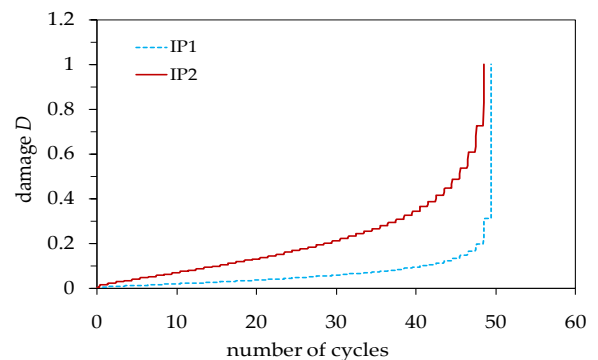


FIG. 12 DAMAGE EVOLUTION IN TWO INTEGRATION POINTS

The damage evolving in IP2 is faster than in IP1. This is correct because IP2 is closer to the crack tip. During the crack growth, the failure sequence is from IP2 to IP1. So, the damage level in IP2 is always higher. The other feature in Fig.12 should also be paid attention. The damage accumulation in cohesive element has a ladder-shaped distribution. The reason is that the damage does not evolve in unloading and a part of reloading process.

The fatigue crack growth rates curve is the key point for comparison. The fatigue crack growth rate $\Delta a/\Delta N$ can be calculated directly, but the cyclic ΔJ -integral will be computed by another approach. A finite element model of CT specimen without cohesive zone is built. The crack length inserted in this model is $(a_0 + \Delta a)$. For one cycle the cyclic energy integral ΔJ can be computed by equation (13) (Wüthrich 1982).

$$\Delta J = \int_{\Gamma} \left(\Delta W dy - \Delta t_i \frac{\partial \Delta u_i}{\partial x} ds \right)$$

$$\Delta W = \int_0^{\Delta \varepsilon} \Delta \sigma_{ij} d\Delta \varepsilon_{ij} \quad (13)$$

$\Delta \sigma_{ij}$ and $\Delta \varepsilon_{ij}$ are cyclic stress and strain range, respectively. ΔW is the cyclic strain energy density. x and y are the Cartesian coordinates with the x -axis parallel to the crack surface. Γ is the integration path. ds is the line element lying on the integration path. Δt_i is the cyclic stress vector on the integration path. Δu_i is the cyclic displacement variation.

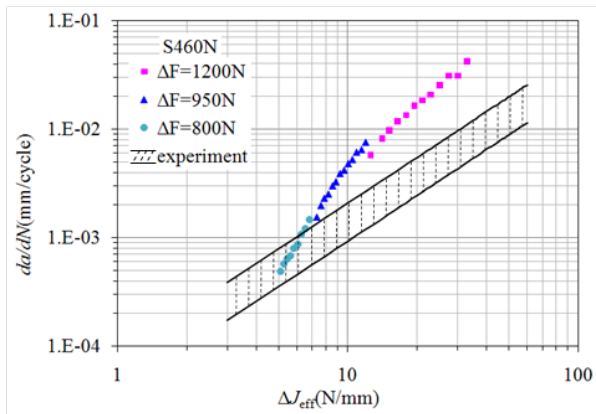


FIG. 13 COMPARISON OF FATIGUE CRACK GROWTH RATES CURVE FOR S460N

The simulation results under three loading ranges are plotted together with the scatter band of experimental data in Fig. 13. The experimental Paris law can be reproduced by the CZM simulation to some extent. The deviation between experiment and simulation can also be accepted. The simulated crack growth rates curve seems to enter into an unstable region of fatigue

crack growth.

The simulated fatigue crack growth is faster than the experiment. One possible reason for this deviation is from the influence of the model dimension. In the experiment, the specimen is a three dimensional model, but in the simulation a two dimensional model is used. The calculation of the ΔJ -integral leads to larger values for two dimensional problems than for three dimensional problems. Besides, the CZM may express the real cyclic process inaccurately. In the future more effort will be put into the improvement of the model.

Conclusion and Outlook

As a tool for material monotonic fracture analysis, the CZM is developed for many years and now it becomes a mature model. But for fatigue investigation, the CZM is just in a beginning state. In this paper, taking an idea from the pioneer as reference, a cyclic CZM including a new damage evolution law is proposed. This CZM involves the triaxiality influence characteristic and can be applied for various triaxiality conditions. In the very low cycle fatigue regime, the triaxiality dependent cyclic CZM is employed for fatigue crack initiation and fatigue crack growth simulation, reasonable agreement between experiment and simulation can be obtained.

The new cyclic CZM is a developing numerical model and many improvements can be done in the future. The format of the damage evolution law may be modified. The triaxiality considered presently has a range boundary $1 \leq H \leq 4$. Using GTN model simulations this range can be extended. The triaxiality dependent behavior for unloading/compression is still a blank research field. If using the method in this paper, the triaxiality value keeps constant in the unloading process and a part of the reloading process. The triaxiality in the whole cyclic process will be discontinuous. This discontinuous triaxiality will lead to convergence problems. The triaxiality dependence characteristic for cohesive parameters is just taken from GTN model simulation. In reality, the triaxiality influence for cohesive parameters may be weaker. So, the correlative experiments need to be carried out in the future.

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